

A NOTE ON CONSTRUCTION OF TRIANGULAR PBIB DESIGN WITH PARAMETERS

$$v=21, b=35, r=10, k=6, \lambda_1=2, \lambda_2=3$$

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The triangular PBIB design with parameters $v=21, b=35, r=10, k=6, \lambda_1=2, \lambda_2=3$ is listed in Chang, Liu and Liu (1965) as one of the unsolved cases. Aggarwal (1972) has given the solution of this triangular design.

His method is set out below :

The PBIB design with parameters $v=21, b=42, r=12, k=6, \lambda=3$ is obtained from initial sets $(0_1, 5_1, 1_2, 4_2, 2_3, 3_3), (0_1, 1_1, 3_1, 0_2, 1_2, 3_2), (0_2, 5_2, 1_3, 4_3, 2_1, 3_1), (0_2, 1_2, 3_2, 0_3, 1_3, 3_3), (0_3, 5_3, 1_1, 4_1, 2_2, 3_2)$ and $(0_3, 1_3, 3_3, 0_1, 1_1, 3_1) \pmod{7}$. This solution is listed in Raghavarao (1971). Aggarwal (1972) observed that the first set when generated gives a triangular PBIB design with parameters $v=21, b=7, r=2, k=6, \lambda_1=1, \lambda_2=0$. If these sets are deleted from the sets of the BIB design, one gets the triangular PBIB design with parameters $v=21, b=35, r=10, k=6, \lambda_1=2, \lambda_2=3$.

It may easily be verified that two more initial sets viz., $(0_2, 5_2, 1_3, 4_3, 2_1, 3_1)$ and $(0_3, 5_3, 1_1, 4_1, 2_2, 3_2)$ when generated each give the triangular PBIB design $v=21, b=7, r=2, k=6, \lambda_1=1, \lambda_2=0$. One thus finds that the solution of the PBIB design with $v=21, b=35, r=10, k=6, \lambda_1=2, \lambda_2=3$ can be obtained by deleting any of the three sets $(0_1, 5_1, 1_2, 4_2, 2_3, 3_3), (0_2, 5_2, 1_3, 4_3, 2_1, 3_1)$ or $(0_3, 5_3, 1_1, 4_1, 2_2, 3_2)$.

It can also be noted that if any two sets out of the above three are combined they do not form any meaningful design. Thus by deleting two sets at a time no PBIB design can be obtained. Similarly, it can be seen that by deleting all the above said three sets no PBIB design can be constructed.

APPENDIX

Design matrix for $v=5, 8$ are given below—

$v=5$

	3	4	5	2
3		5	1	4
5	4		2	1
2	5	1		3
4	1	2	3	

$v=8$

	3	4	5	6	7	8	2
3		5	6	7	8	1	4
5	4		7	8	1	2	6
8	5	6		1	2	3	7
2	6	7	8		3	4	1
4	7	8	1	2		5	3
6	8	1	2	3	4		5
7	1	2	3	4	5	6	

REFERENCES

- Aggarwal, K.R. (1972) : A note on construction of triangular PBIB design with parameters $v=21$, $b=35$, $r=10$, $k=6$, $\lambda_1=2$, $\lambda_2=3$. Annals Maths. Statistics, 43, 371.
- Chang, L.C. Liu, C.W.
and Liu, W.R. (1965) : Incomplete block designs with triangular parameters for which $k=10$ and $r=10$. Scientia Sinsia, 14, 329-338.
- Raghavarao, D. (1971) : Constructions and Combinatorial problems in Design of Experiment, John. Willy and Sons.